Mad Abel: A card game for 2+ players

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Perhaps it is due to rather than despite the fact that I have no time to sit around and play games that I came up with one. It is not easy to play this game quickly, and takes a fair bit of practice, but it is quite entertaining and provides multiple variants. Also, due to it's basis in abstract algebra - specificly Abelian groups of prime order with addition - it's a fairly good excercise in addition under unusual circumstances, and can be changed in any number of ways to make it even more complicated. In it's simplest form, it can be used to geeky card players to show off their moduluo-calculation skills, or in a high school classroom as a fun intro to groups and or modulus. In it's more complex forms, university level math students can show off the best of their abilities in various states of intoxication.

1 Game rules

Each player is dealt seven cards, and two cards are placed face up on the table. The two cards on the table are summed together according to a pattern described later. The sum is the card requested. Each player takes turns placing down one or more cards that add up to the top two cards on the pile. If a player is unable to create the correct card from cards on hand, he must draw two cards from the deck and pass the game over to the next player. The player that empties his hand first is the winner.

The summation rule is based on the order $\heartsuit, \spadesuit, \diamondsuit, \clubsuit$, where one could, for simplicity, make the equations $\heartsuit := 0, \spadesuit := 1, \diamondsuit := 2, \clubsuit := 3$, and do all arithmetic modulo 4. The Cayley table of which is:

$+$	$\heartsuit$	$\spadesuit$	$\diamondsuit$	$\clubsuit$
$\heartsuit$	$\heartsuit$	$\spadesuit$	$\diamondsuit$	$\clubsuit$
$\spadesuit$	$\spadesuit$	$\diamondsuit$	$\clubsuit$	$\heartsuit$
$\diamondsuit$	$\diamondsuit$	$\clubsuit$	$\heartsuit$	$\spadesuit$
$\clubsuit$	$\clubsuit$	$\heartsuit$	$\spadesuit$	$\diamondsuit$

A useful feature of the $\heartsuit, \spadesuit, \diamondsuit, \clubsuit$, which is common in Iceland but not traditional in English speaking countries ($\spadesuit, \heartsuit, \diamondsuit, \clubsuit$, being the cannonical ordering there) is that "same colors make red, different colours make black." That is to say, if you sum up a red and a red or a black and a black, the result will be red, and if a red and black are summed, the result will be black. Compare to the "plus-minus" rule.

The numbers also have to be summed, mod 13. This means that we can define $K := 0$, $Q := 12$, $J := 11$ and $A := 1$. Some people find it comfortable to bear in mind that $Q \equiv -1$, $J \equiv -2$, and even $10 \equiv -3$. Using K as the identity preserves the ordering, but people wanting a little bit of insanity can of course relabel their set.

The Cayley table for this is very typical:

$+$	K	A	2	3	4	5	6	7	8	9	10	J	Q
K	K	A	2	3	4	5	6	7	8	9	10	J	Q
A	A	2	3	4	5	6	7	8	9	10	J	Q	K
2	2	3	4	5	6	7	8	9	10	J	Q	K	A
3	3	4	5	6	7	8	9	10	J	Q	K	A	2
4	4	5	6	7	8	9	10	J	Q	K	A	2	3
5	5	6	7	8	9	10	J	Q	K	A	2	3	4
6	6	7	8	9	10	J	Q	K	A	2	3	4	5
7	7	8	9	10	J	Q	K	A	2	3	4	5	6
8	8	9	10	J	Q	K	A	2	3	4	5	6	7
9	9	10	J	Q	K	A	2	3	4	5	6	7	8
10	10	J	Q	K	A	2	3	4	5	6	7	8	9
J	J	Q	K	A	2	3	4	5	6	7	8	9	10
Q	Q	K	A	2	3	4	5	6	7	8	9	10	J

Don't worry if it seems big and daunting. You'll get used to it very quickly.

2 Example game

Now, as an example of how it all works, let's set up an example game and do the first few rounds.

We are dealt seven cards: $\spadesuit 5, \spadesuit 8, \spadesuit 9, \spadesuit J, \spadesuit K, \diamondsuit 6, \clubsuit 8$. Our imaginary opponent gets dealt seven cards - we shall not be privy to their values. The table shows $\heartsuit 10, \diamondsuit 9$.

$$\heartsuit 10 + \diamondsuit 9 = \diamondsuit 6$$

So that's the card we shall put down. Lucky we have that on hand, so we place it on top of the $\heartsuit 10$, leaving $\diamondsuit 6, \heartsuit 10$ on top.

$$\diamondsuit 6 + \heartsuit 10 = \diamondsuit 3$$

Our opponent grumbles to himself and thinks for a moment. Then he smiles and lays down $\spadesuit 6 + \spadesuit 10$. What just happened? Well, as you can see,

$$\spadesuit 6 + \spadesuit 10 = \diamondsuit 3$$

So what he did was legal. What's more, because his combination was exactly two cards, we now have to place down the same card as he just did! $\diamondsuit 3$! We look over our cards and see that we are blessed with another way of making $\diamondsuit 3$: $\spadesuit 9 + \spadesuit 5$. Now, this is getting boring, for $\diamondsuit 3$ is helping noone at present, so we look over our cards and see that we can also make a diamond by way of the $\clubsuit 8$ plus any spade, and then adding two spades to it. However, a bit of arithmetic shows us that none of our combinations of four cards would give us the $\diamondsuit 3$, so we're forced to play out the pair.

Our opponent swears quite rudely and draws two cards. Since we no longer have any way of forming $\diamondsuit 3$ either, we draw two too - $\diamondsuit K$ and, remarkably, $\diamondsuit 3$[1].

At this point, just as we are silently gloating that we have the card we want, our opponent says "har har" in a very piratelike voice, and puts down three cards: $\clubsuit Q, \spadesuit 7, \diamondsuit 10$. We look it over, and rightly enough:

$$(\clubsuit Q + \spadesuit 7) + \diamondsuit 10 = \heartsuit 6 + \diamondsuit 10 = \diamondsuit 3$$

Now, he could have placed his three cards down in any order he wanted, and indeed there are three possible orderings (since the groups are Abelian, the order of a pair does not matter, dividing our sorting options in two). He opted to let $\clubsuit Q, \spadesuit 7$ face up, and since we do not have $\heartsuit 6$ on hand, we can assume that he either has a way of creating $\heartsuit 6$ on hand, the card itsself, or no method of creating $\heartsuit 6, \spadesuit 9$ or $\clubsuit 4$ - his three options. This might be able to help us, although at the moment we are more fussed by the fact that we, indeed, have to come up with a $\heartsuit 6$.

[1]The reader may be asking himself at this time if this game was real or rigged. However, it was real insofar as cards were dealt and drawn; however I played both sides in this particular instance.

Our hand at this point is $\spadesuit 8, \spadesuit J, \spadesuit K, \diamondsuit 3, \diamondsuit K, \clubsuit 8$, Immediately we see that $\diamondsuit K + \spadesuit J + \spadesuit 8 = \heartsuit 6$. Likewise we see that we have $\clubsuit 8$ on hand, so placing $\diamondsuit K, \spadesuit 8$ at the top is quite prudent, in case our opponent can't make his move.

He grimmaces at his four cards on hand, sees that it is in vain, and draws two cards. Indeed we were lucky! We play out our $\clubsuit 8$, leaving us with only two cards. Alas, at this point it becomes tricky, because the fewer cards one has, the harder it is to create any card that pops up. Due to this fact, a single game can go on for quite some time.

Our opponent is faced with $\clubsuit 8, \diamondsuit K$, amounting to $\spadesuit 8$. We know that he cannot resolve that with one card, because we used that card in the previous move. But he is not without hope - he thinks for a moment and then puts down three cards: $\heartsuit Q, \clubsuit 10, \diamondsuit Q$.

Alas, this could go on forever... so rather than bore you with the details of how I, three turns later, lost to my imaginary opponent, let's talk about some variants.

3 Prime variants

You can make very fun variants with 11, 7 or 5 cards per suit - any less than that becomes very dull. Fewer cards make less options and thus a shorter game. Rather than go on and on about it, I'll just give you the Cayley tables and you can try for yourself. The only thing that really changes is that the highest card is always the zero card. You'll figure it out.

My personal favourite is the 5-variant. It's quite quickly played

3.1 5-variant

+	5	A	2	3	4
5	5	A	2	3	4
A	A	2	3	4	5
2	2	3	4	5	A
3	3	4	5	A	2
4	4	5	A	2	3

3.2 7-variant

+	7	A	2	3	4	5	6
7	7	A	2	3	4	5	6
A	A	2	3	4	5	6	7
2	2	3	4	5	6	7	A
3	3	4	5	6	7	A	2
4	4	5	6	7	A	2	3
5	5	6	7	A	2	3	4
6	6	7	A	2	3	4	5

3.3 11-variant

+	J	A	2	3	4	5	6	7	8	9	10
J	J	A	2	3	4	5	6	7	8	9	10
A	A	2	3	4	5	6	7	8	9	10	J
2	2	3	4	5	6	7	8	9	10	J	A
3	3	4	5	6	7	8	9	10	J	A	2
4	4	5	6	7	8	9	10	J	A	2	3
5	5	6	7	8	9	10	J	A	2	3	4
6	6	7	8	9	10	J	A	2	3	4	5
7	7	8	9	10	J	A	2	3	4	5	6
8	8	9	10	J	A	2	3	4	5	6	7
9	9	10	J	A	2	3	4	5	6	7	8
10	10	J	A	2	3	4	5	6	7	8	9

4 Other variants

There's plenty of other variations possible. Non-Abelian games can be quite interesting. Just try messing up the numbers a bit, and be careful to use multiplication rather than addition, otherwise you won't get any nonabelian groups. You could also try making some new operators, or even considering each card as a vector and demand a vector of the same length (this might be tricky, depending on which metric you use! I've tried playing the 5-variant with the Taxi Cab metric, and it was confusing but fun).

There are also a number of ways you can play this as a single player game, but I've not found any very fun ones yet.

If you find any particularly fun variants, by all means let me know via e-mail.